

$G = \langle x \rangle$  &  $G$  is abelian.

Let  $x, y \in G$ .

$$\exists \phi: G \rightarrow H \quad \phi(xy) = \phi(x)\phi(y) \quad \forall x, y \in G.$$

$G$  is cyclic  $\Rightarrow G$  is abelian.

Let  $n, m \in H$ .

Good!

Prove that  $nm = mn$ .

Let  $\phi$  be the isomorphic map  $G \rightarrow H$ .  $\phi$  is onto, by def. of isomorphism.

$\exists x \in G \mid \phi(x) = n$  since  $\phi$  is onto.

$\exists y \in G \mid \phi(y) = m$  " \_\_\_\_\_".

$$\begin{aligned} \text{So, } nm &= \phi(x)\phi(y) \\ &= \phi(xy) \quad \checkmark \quad \text{b/c } \phi \text{ is a } \text{homomorphism} \\ &= \phi(yx) \quad \text{b/c } G \text{ is } \text{cyclic \& abelian.} \\ &= \phi(y)\phi(x) \quad \text{b/c } \phi \text{ is a } \text{homomorphism.} \\ &= mn \quad \checkmark \end{aligned}$$

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