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5. Proof: Let $x \in H$ and $y \in H$. Show $xy = yx$.

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$\because$ G is isomorphic to H.

$\therefore$ there exist $f: G \to H$ such that f is an isomorphism, which means ① f is an homomorphism; ② f is 1-1; ③ f is onto.

$\because$ f is onto, so $\exists\, a \in G$ s.t $f(a) = x$; likewise, $\exists\, b \in G$, s.t $f(b) = y$

and $\because$ f is an homomorphism

$\therefore$ $f(ab) = f(a) f(b)$ $(a, b \in G)$

$\because$ G is cyclic $\therefore$ G is abelian $\Rightarrow$ $ab = ba$

So we have $xy = f(a) f(b) = f(ab) = f(ba) = f(b) f(a) = yx$

$\therefore$ $xy = yx$ for $x, y \in H$

$\therefore$ H is abelian.

Well-written!