Test #1

5) 20

Proof: Let $a, b \in H$. ~~to~~ To show: $ab = ba$.

Because $G \simeq H$, $\exists\, \phi : G \xrightarrow[\text{onto}]{1-1} H$

s.t. $\phi(\alpha\beta) = \phi(\alpha)\phi(\beta)$ for all $\alpha, \beta \in G$.

Since ϕ is onto, $\exists\, \alpha_0, \beta_0 \in G$ s.t. $a = \phi(\alpha_0)$ and $b = \phi(\beta_0)$. We thus have

$$\phi(\alpha_0 \beta_0) = \phi(\alpha_0)\phi(\beta_0) = ab$$

and

$$\phi(\beta_0 \alpha_0) = \phi(\beta_0)\phi(\alpha_0) = ba$$

But G is abelian since it is cyclic, which means that $\alpha_0\beta_0 = \beta_0\alpha_0$. It follows that $\phi(\alpha_0\beta_0) = \phi(\beta_0\alpha_0)$, or $ab = ba$ //

Well-written!