Math 455 Winter 2007 Test #1

Each question is worth 20 points. Justify all of your answers!

#1) Let $G = \mathbb{R}$ be the group of real numbers under addition.

a. Let $H = \{x \in \mathbb{R} \mid x \geq 0\}$. Is H a subgroup of G?

b. Let $K = \{a + b\sqrt{5} \mid a, b \in \mathbb{Z}\}$. Is K a subgroup of G?

#2) Find the order of the element 80 in the additive group $\mathbb{Z}_{200}$.

#3) Define $f, g, h, \in S_5$ by:

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 2 & 1 \end{pmatrix} \qquad g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 5 & 3 & 4 \end{pmatrix} \qquad h = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 1 & 2 & 3 \end{pmatrix}$$

Find $x \in S_5$ such that $fxg = h$.

#4) Cayley tables for three groups G, H, and K are given.

a. Exactly one of these three groups is nonabelian. Which one? Justify your answer. (You do NOT need to justify that the other two are abelian.)

b. Exactly one of these three groups is cyclic. Which one? Justify your answer. (You do NOT need to justify the fact that neither the other two are cyclic.)

$\underline{G}$

	h	i	j	k	ℓ	m	n
h	h	i	j	k	ℓ	m	n
i	i	j	k	ℓ	m	n	h
j	j	k	ℓ	m	n	h	i
k	k	ℓ	m	n	h	i	j
ℓ	ℓ	m	n	h	i	j	k
m	m	n	h	i	j	k	ℓ
n	n	h	i	j	k	ℓ	m

$\underline{H}$

	p	q	r	s	t	u	v	w
p	p	q	r	s	t	u	v	w
q	q	p	s	r	u	t	w	v
r	r	s	q	p	v	w	u	t
s	s	r	p	q	w	v	t	u
t	t	u	w	v	q	p	r	s
u	u	t	v	w	p	q	s	r
v	v	w	t	u	s	r	q	p
w	w	v	u	t	r	s	p	q

$\underline{K}$

	a	b	c	d	f	g	x	y	z
a	a	b	c	d	f	g	x	y	z
b	b	c	a	f	g	d	y	z	x
c	c	a	b	g	d	f	z	x	y
d	d	f	g	x	y	z	a	b	c
f	f	g	d	y	z	x	b	c	a
g	g	d	f	z	x	y	c	a	b
x	x	y	z	a	b	c	d	f	g
y	y	z	x	b	c	a	f	g	d
z	z	x	y	c	a	b	g	d	f

#5) Prove in gory detail that if G is a cyclic group and H is a group and G is isomorphic to H, then H is abelian. (A hint is available upon request. The hint will cost you 10 points. It is meant to get you started if you are completely stuck.)

Hint: Since G is cyclic, we know that there exists $a \in G$ such that $G = \langle a \rangle = \{a^n \mid n \in \mathbb{Z}\}$. Since G is isomorphic to H, we know that there exists a function $f : G \to H$ such that f is a homomorphism, f is one-to-one, and f is onto. Let $b = f(a)$. As a first step in this problem, prove that $H = \langle b \rangle$. Hopefully, this should get you unstuck.

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