Section 9 – Problem #23.

1. Some permutations are not cycles.

True. For example, (1,2)(3,4) is not a cycle. But every permutation is a product of disjoint cycles.

2. Every cycle is a permutation.

True.

3. Every nontrivial subgroup H of S_9 containing some odd permutation contains a transposition.

False. For a counterexample, let H be the subgroup generated by (1,2,3,4).

4. A_5 has $5! / 2 = 60$ elements.

True.

5. S_n is not cyclic for any $n \geq 1$ ($n \geq 2$ to be cyclic)

A true statement is "S_n is not cyclic for any $n > 2$." (You showed in a previous HW that S_n is not abelian for any $n > 2$. And if it's not abelian, then it can't be cyclic.)

6. A_3 is a commutative group.

True. (In fact, you can use Lagrange's Theorem to prove it is cyclic.)

7. S_7 is **not** isomorphic to the subgroup of all those elements of S_8 that leave the number 8 fixed.

False. In fact, S_7 is isomorphic to the subgroup of all those elements of S_8 that leave the number 8 fixed. To see this, make the following definitions:

If s is in S_7, then define s' from {1,2,3,4,5,6,7,8} to {1,2,3,4,5,6,7,8} by s'(n)=s(n) if n<8 and s'(8)=8.

Let H be the subgroup of all those elements of S_8 that leave the number 8 fixed.

Define f from S_7 to H by f(s)=s'.

Then show that f is an isomorphism.

8. S_7 **is** isomorphic to the subgroup of all those elements of S_8 that leave the number 5 fixed.

True. (Kinda like #7.)

9. The odd permutations in S_8 form a subgroup of S_8.

False. Let A be the subset of all odd permutations in S_8. Then A is not closed under the group law. For example, (1,2) and (3,4) are in A, but their product is not.