Section 13 – Problem 32.

1. A_n is a normal subgroup of Sn.

True.

2. For some groups G and G' there exists a homomorphism of G into G'.
[It is not true that for **any** 2 groups there exists a homomorphism of G into G'.]

True.

3. Some homomorphisms are not a one-to-one map.
[A group homomorphism ϕ :G $\to$ G' is a 1-to-1 map iff Ker [ϕ] = {e}.]

No need to change this one; it was true in the first place. (What you've written is also true.)

4. A homomorphism is one to one iff the kernel consists of the identity element alone.

True.

5. The image of a group of 6 elements under some homomorphism **cannot have** 4 elements.
[Let ϕ :G $\to$ G', be a group homomorphism, then if | G | is finite, then | ϕ [G] | is finite and is a divisor of | G |, and if | G' | is finite, then | ϕ [G] | is finite and is a divisor of | G' |, but 4 is not a divisor of 6, then the image cannot have 4 elements].

Correct. Did you need the fact that | ϕ [G] | is finite and is a divisor of | G' |.

6. The image of a group of 6 elements under some homomorphism may have 12 elements.

False. You can't map 6 things to 12 things.

7. There is a homomorphism of some group of 6 elements into some group of 12 elements.

True. For example, define f : Z_6 to Z_12 by f(n)=2n. (Check that f is a well-defined homomorphism.)

8. There is **no** homomorphism of some group of 6 elements into some group of 10 elements.

False. There is always the trivial homomorphism that maps everything to zero.

9. A homomorphism **cannot have** an empty kernel.

True. A kernel always contains the identity.

10. **It is possible** to have a nontrivial homomorphism of some finite group into some infinite group.

True. For example, define f : Z_2 to Z by f(n)=0.