

Section 11 – Problems #32 and 36. (page 112)

#32

1. If G_1 and G_2 are any groups $G_1 \times G_2$ is not always isomorphic to $G_2 \times G_1$.

False. This is one of the problems on your review sheet.

2. $Z_2 \times Z_4$ is not isomorphic to Z_8 .

True.

3. $Z_2 \times Z_4$ is isomorphic to S_8 .

False. $Z_2 \times Z_4$ has order 8. S_8 has order $8!$

4. $Z_3 \times Z_8$ is not isomorphic to Z_4 .

True. It is also true that $Z_3 \times Z_8$ is not isomorphic to Z_{24} .

5. Every element in $Z_4 \times Z_8$ has order 8.

False. Consider $(1,0)$. It is true that if (a,b) is in $Z_4 \times Z_8$, then $8(a,b)=(0,0)$.

6. The order of $Z_{12} \times Z_{16}$ is 60 [$\text{lcm}(12, 16) = 2 \cdot 2 \cdot 3 \cdot 5 = 60$]

False. The order of $Z_{12} \times Z_{16}$ is $12 \times 16 = 192$. The order of $(1,1)$ in $Z_{12} \times Z_{16}$ is 60.

7. $Z_m \times Z_n$ has mn elements **only** if m and n are relatively prime.

False. $Z_m \times Z_n$ has mn elements **whether or not** m and n are relatively prime

#36

1. Every abelian group of prime order is cyclic.

True. In fact, every group of prime order is cyclic.

2. Abelian group of prime power order **are not** cyclic.

To make a true statement, replace “are not” with “may or may not be.”

3. Z_8 is generated by $\{4, 6\}$.

False. 1 is not in the subgroup generated by 4 and 6.

4. \mathbb{Z}_8 is not generated by $\{4, 5, 6\}$.

False. Let m in \mathbb{Z}_8 . Then $m = m \times 5 - m \times 4$, which is in the subgroup generated by 5 and 4.

5. All finite abelian groups are classified up to isomorphism by the Fundamental Theorem of Finitely Generated Abelian Groups.

6. Every abelian group of order divisible by 5 **does not** contains a cyclic subgroup of order 5.

7. Every abelian group of order divisible by 4 contains a cyclic subgroup of order 4.

8. Every abelian group of order divisible by 6 contains a cyclic subgroup of order 6.

For the test, you don't have to worry about these four.